

A novel method of computing the Stess intensity factors of the interfacial cracks

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A noval method of computing the Stess intensity factors of the interfacial cracks

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Abstract—Bi-material interface and multi-layer systems are widely observed in modern microelectronic applications. When the external load reaches a critical level, the crack either extends along the interface or kinks out of the interface, and finally leads to the catastrophic failure. In fracture mechanics, stress intensity factor, mode mix ratio and strain energy release rate are normally used as parameters to evaluate the adhesive toughness and failure prediction of bi-material interfaces. In this research, a new efficient method based on the finite elements and the extended proportional method using nodal-displacement behind the crack tip was introduced to obtain the stress intensity factors, then the strain energy release rate could be computed by using its relationship with the stress intensity factors. The robustness and accuracy of the current proposed method was discussed by comparing the solution results proposed by other researchers. It was found that the average error is less than 1% for the stress intensity factors, and it can get accurate results with rather coarse finite element meshes. Furthermore, the current method is fairly efficient and less computational resource consuming. The current method could be used as an effective tool in the reliability analysis of the bonded multi-layers in microelectronics.

Keywords—stress intensity factor; mode mix ratio; finite element method;

I. INTRODUCTION

Teranishi and Nisitani [1] firstly proposed a numerical procedure "Crack Tip Stress Method" for determining the SIFs of the homogenous crack problems by using FE Method. In this procedure, the SIF of a target unknown problem is solved by using the ratio of the crack tip stresses of the reference and the target unknown problems computed by FE method. Then, Oda et al [2] extended this method to analyze the interface crack problems by making the singular terms the same for the reference and target unknown problems. Lan et al [3-4] solved the SIFs of several edge interface cracks in bi-material bonded strips for arbitrary material combinations based on Oda's method. Noda and Lan [5] investigated the method proposed by Oda systematically and proposed a linear extrapolation technology to improve the accuracy. However, the methods above need very refined meshes to improve the accuracy and are too much time consuming. In this research, the authors

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proposed a noval method based on the ratio of the relative crack opening displacement of the FE nodes behind the crack tip. As a result, the current method gives reliable results with rather coarse FE meshes. Meanwhile, the computational efficiency of the current method is significantly improved by using less computational resource.

II. ANSLYSIS METHOD

A. The physical backgroud of the crack tip stress method

According to the theory of linear-elastic fracture mechanics (LEFM), mode I SIF near the crack tip in a homogenous plate shown in Fig.1 is defined by the following equation.

$$\sigma_{y}(r) \rightarrow \frac{K_{I}}{\sqrt{2\pi r}}, \quad \sigma_{y} = \sigma_{yy}|_{\theta=0} (r \rightarrow 0)$$
 (1)

Here, K_I is the mode I SIF, σ_{yy} is the normal stress component ahead of the crack tip. For a given point at $\theta = 0$ with a distance from the crack tip $r = r_0$, $K_I/\sigma_y = \sqrt{2\pi r_0}$ is constant and a following relationship can be deduced theoretically for two different crack problems A and B.

$$\left[K_{I}^{*}/\sigma_{y}^{*}\right]_{A} = \left[K_{I}/\sigma_{y}\right]_{B}$$
(2)

Assuming the SIF for problem A is analytically given in advance, while that for problem B is yet to be solved. Problem A is denoted as the reference problem and problem B is denoted as the given unknown problem. Here, the superscript * is introduced to indicate the values of the reference problem A for notational convenience. Although the true values of σ_y^*, σ_y in (2) cannot be computed by FE analysis due to the crack tip singularity, the ratio of the value can be given accurately without difficulty. This is because the error for the problems A and B are nearly the same if the same FEM meshes are applied to the problem B can be solved by using the ratio of the crack tip stresses in (2) computed by FE method. This is the physical background of "Crack Tip Stress Method". In this paper the theory will be extended to the interface crack case.



Fig.1 Normal stress distribution ahead of the crack tip

B. Formulations for the interfrace crack problems

The associated crack flank displacements at a distance r behind the crack tip δ_{m} shown in Fig.1 is defined as

$$\delta_m = u_m (r, \theta = \pi) - u_m (r, \theta = -\pi), (m = x, y)$$
(3)

The values δ_{y}, δ_{x} are given by

$$\delta_{y} + i\delta_{x} = \frac{K_{I} + iK_{II}}{2(1+2i\varepsilon)\cosh(\varepsilon\pi)} \left[\frac{\kappa_{1}+1}{\mu_{1}} + \frac{\kappa_{2}+1}{\mu_{2}}\right] \left(\frac{r}{2\pi}\right)^{1/2} \left(\frac{r}{l}\right)^{i\varepsilon}$$
(4)

where $\mu_m (m=1,2)$ and $\nu_m (m=1,2)$ are the shear moduli and Poisson's ratios of either respective materials. *r* is the radial distance behind the crack tip, *l* is an arbitrary reference length which scales with specimen size or crack length, and ε is the bi-elastic constant given by:

$$\varepsilon = \frac{1}{2\pi} \ln \left[\left(\frac{\kappa_1}{\mu_1} + \frac{1}{\mu_2} \right) \middle/ \left(\frac{\kappa_2}{\mu_2} + \frac{1}{\mu_1} \right) \right]$$
(5)

$$\kappa_{m} = \begin{cases} 3 - 4\nu_{m} \quad (plane \ strain) \\ 3 - \nu_{m}/1 + \nu_{m} \quad (plane \ stress) \end{cases}$$
(6)

Considering $(r/l)^{i\varepsilon} = \cos(\varepsilon \ln(r/l)) + i \sin(\varepsilon \ln(r/l))$ and rearranging (4), then the SIF components K_i, K_{ii} can be separated as :

$$\frac{K_{I}}{\delta_{y}} = A \left\{ \left(\cos Q + 2\varepsilon \sin Q \right) + \left(\sin Q - 2\varepsilon \cos Q \right) \frac{\delta_{x}}{\delta_{y}} \right\}$$
(7)

$$\frac{K_{II}}{\delta_x} = A \left\{ \left(\cos Q + 2\varepsilon \sin Q \right) - \left(\sin Q - 2\varepsilon \cos Q \right) \frac{\delta_y}{\delta_x} \right\}$$
(8)

$$A = \frac{2\cosh\left(\varepsilon\pi\right)\left(r/2\pi\right)^{-1/2}}{\left(\frac{\kappa_1+1}{\mu} + \frac{\kappa_2+1}{\mu_2}\right)}$$
(9)

$$Q = \varepsilon \ln(r/l) \tag{10}$$

From (7) and (8), when $Q, \delta_y/\delta_x$ are kept the same for different cracks, we get a similar proportional relationship as the homogenous crack case in the following

$$K_I / \delta_y = const, K_{II} / \delta_x = const$$
 (11)



Fig.2 Central cracked dissimilar bonded half-planes (The reference problem C)

Equation (11) depicts that the theory of the crack tip stress method [1] could be extended to the interfacial cracks using the proportional ratio of the crack opening displacements.

C. Application of the finite element method

Let's consider two interfacial crack problems C and D. Problem C is the reference whose SIFs are known in advance, and problem D is the target unknown problem whose SIFs are waiting to be solved. Giving the following two conditions satisfied

$$\left[Q^*\right]_c = \left[Q^*\right]_D \tag{12}$$

$$\left[\delta_{y}^{*}/\delta_{x}^{*}\right]_{C} = \left[\delta_{y}/\delta_{x}\right]_{D}$$
(13)

Using (11) into problems C and D, then we have

$$\frac{K_{I}}{K_{I}} = \frac{\left\lfloor \delta_{y} \right\rfloor_{D}}{\left\lfloor \delta_{y}^{*} \right\rfloor_{C}} = \frac{\left\lfloor \delta_{y,FEM} \right\rfloor_{D}}{\left\lfloor \delta_{y,FEM}^{*} \right\rfloor_{C}}, \frac{\left\lfloor K_{II} \right\rfloor_{D}}{\left\lfloor K_{II}^{*} \right\rfloor_{C}} = \frac{\left\lfloor \delta_{x} \right\rfloor_{D}}{\left\lfloor \delta_{x}^{*} \right\rfloor_{C}} = \frac{\left\lfloor \delta_{x,FEM} \right\rfloor_{D}}{\left\lfloor \delta_{x,FEM}^{*} \right\rfloor_{C}}$$
(14)

Where asterisk (*) is employed to denote the parameters for the reference problem. In (14), the SIFs of the reference (problem C) are theoretically known in advance, and the relative crack opening displacements $\delta_{y,FEM}$, $\delta_{x,FEM}$ can be obtained using FE analysis. Then the SIFs of the given unknown problem (problem D) can be easily solved using (14).

The reference problem C considered here is a finite center crack between two dissimilar isotropic materials in an infinite panel subject to both remotely uniform normal and shear stresses as shown in Fig.2. The analytical solution was firstly theoretically derived by Rice and Sih (1965) and takes the form

$$K_{I}^{*} + iK_{II}^{*} = \left(\sigma_{y}^{\infty} + i\tau_{xy}^{\infty}\right)\sqrt{\pi a}\left(1 + 2i\varepsilon\right)$$
(15)

Here, σ_y^{∞} , τ_{xy}^{∞} are the remote uniform tension and shear applied to the bonded half-planes. *a* is the half crack length of the center crack. The prerequisite (12) can be easily satisfied by applying the same material combinations ε_0 and investigating the nodes of the same relative distances behind the crack tip r_0 for the two problems. In the following we will derive how to make the prerequisite (13) satisfied.

The stress components of the reference problem C can be solved in an indirect manner using the principle of superposition. The reference problem can be solved in two steps (pure remote tension $\sigma_y^{\infty} = T, \tau_{xy}^{\infty} = 0$ and pure remote shear $\sigma_y^{\infty} = 0, \tau_{xy}^{\infty} = S$). Let $\delta_{x,FEM}^{(\sigma_y^{\infty}=1,\tau_{xy}^{\infty}=0)^*}, \delta_{y,FEM}^{(\sigma_y^{\infty}=1,\tau_{xy}^{\infty}=0)^*}$ and $\delta_{x,FEM}^{(\sigma_y^{\infty}=0,\tau_{xy}^{\infty}=1)^*}, \delta_{y,FEM}^{(\sigma_y^{\infty}=0,\tau_{xy}^{\infty}=1)^*}$ denote the crack displacements for the pure remote tension case and the pure remote shear case, respectively. Then we substitute the FE values into (13) and get

$$\frac{S}{T} = \frac{\left[\delta_{x,FEM}\right]_{D} \times \left[\delta_{y,FEM}^{\left(\sigma_{y}^{\varphi}=1,\tau_{xy}^{\varphi}=0\right)^{*}}\right]_{C} - \left[\delta_{y,FEM}\right]_{D} \times \left[\delta_{x,FEM}^{\left(\sigma_{y}^{\varphi}=1,\tau_{xy}^{\varphi}=0\right)^{*}}\right]_{D}}{\left[\delta_{y,FEM}\right]_{D} \times \left[\delta_{x,FEM}^{\left(\sigma_{y}^{\varphi}=0,\tau_{xy}^{\varphi}=1\right)^{*}}\right]_{C} - \left[\delta_{x,FEM}\right]_{D} \times \left[\delta_{y,FEM}^{\left(\sigma_{y}^{\varphi}=0,\tau_{xy}^{\varphi}=1\right)^{*}}\right]_{D}} (16)$$

Applying the loads of S and T shown in (16) to the reference problem, then the SIFs of the target unknown problem D can be obtained using

$$\begin{bmatrix} K_I \end{bmatrix}_D = \frac{\begin{bmatrix} \delta_{y,FEM} \end{bmatrix}_D \begin{bmatrix} K_I^* \end{bmatrix}_C}{\begin{bmatrix} \delta_{y,FEM} * \end{bmatrix}_C}, \begin{bmatrix} K_{II} \end{bmatrix}_D = \frac{\begin{bmatrix} \delta_{x,FEM} \end{bmatrix}_D \begin{bmatrix} K_{II}^* \end{bmatrix}_C}{\begin{bmatrix} \delta_{x,FEM} * \end{bmatrix}_C}$$
(17)

III. NUMERICAL EXAMPLES

The MSC.MARC 2007 r1 finite element analysis package [6] is used to compute the stress components in this research. Fig.3 shows the FE model geometric configurations for the reference problem shown in Fig.2. The crack length for the dissimilar bonded half-planes shown in Fig.3a (the reference) is set to 2a = 2mm in this research. It should be noted that the FE displacement components at the crack tip for the reference problem converge as the width of the model is larger than 1500 times the crack length *a*. Then a length of L = 2W = 6480mm is used to model the reference problem (L = 2W, W/a = 1620). Furthermore, the minimum element size

e of the FE models are kept the same for the reference and the target unknown problems.

The singular regions around the crack tip of both the reference and the target unknown problems are well refined in a self-similar manner. Fig3b shows the FE mesh type in the singular region. The singular region is refined with increasing the number of layers and the element size for each inferior layer is one third of the superior one. The meshes are made of Eight-node quadrilateral elements in plane stress or plane strain condition. Furthermore, the meshes around the crack tip for the reference and target unknown problems are kept the same to make sure a high computational accuracy of $\left[\delta_y^*/\delta_x^*\right]_c = \left[\delta_y/\delta_x\right]_p$.



Fig.3 (a) FE model geometric configurations for the reference problem and (b) the FE mesh in the singular region used for the analysis

A. Central interface cracks

The first example considered here is a central cracked dissimilar bonded strip subjected to uniform normal stress $\sigma = 1$. The crack length 2a is fixed to 2a = 2mm which is the same as the reference problem. The width of the bonded strip W varies from $a/W = 0.1 \sim 0.9$, the length L is assumed to be much greater than the width W (L = 2W is assumed in the FE model). The material properties are $E_1/E_2 = 4$, $v_1 = v_2 = 0.3$. A state of plane stress is assumed. Comparison between the present calculations and those derived by Oda [2], Yuuki and Cho [7] and Miyazaki et al [8] is shown in Table I. For generality, all the SIFs in the table are normalized using the following equation

$$F_{I} = K_{I} / \sigma \sqrt{\pi a}, F_{II} = K_{II} / \sigma \sqrt{\pi a}$$
(18)

It is found that the average error is less than 0.1% for F_I , F_{II} and no very refined finite elements are requested in the computation.

TABLE I.	JORMALIZED SIFS OF THE CENTRAL INTERFACE CRACK
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	Central interface crack						
SIFs	a/W	Present	Ref[2]	Ref[7]	Ref[8]		
FI	0.1	0.987	0.987	0.983	0.987		
	0.2	1.006	1.006	1.005	1.006		
	0.3	1.038	1.038	1.038	1.031		
	0.4	1.088	1.088	1.088	1.089		
	0.5	1.161	1.161	1.162	1.163		
	0.6	1.271	(1.271)	1.272	1.273		
	0.7	1.445	(1.443)	1.445	1.446		
	0.8	1.750	(1.747)	1.751	1.752		
	0.9	2.457	(2.448)	-	-		
FII	0.1	-0.129	-0.129	-0.129	-0.129		
	0.2	-0.127	-0.127	-0.127	-0.127		
	0.3	-0.127	-0.127	-0.127	-0.127		
	0.4	-0.130	-0.130	-0.131	-0.130		
	0.5	-0.138	-0.137	-0.137	-0.137		
	0.6	-0.151	(-0.151)	-0.151	-0.150		
	0.7	-0.177	(-0.176)	-0.176	-0.176		
	0.8	-0.229	(-0.227)	-0.229	-0.227		
	0.9	-0.370	(-0.365)	-	-		

B. Edge interface cracks

The second example is an edge cracked dissimilar bonded strip. The width of the bonded strip W varies from $a/W = 0.1 \sim 0.9$, and the length L is assumed to be 1.5 times the width W. Material properties are $E_1/E_2 = 4$, $v_1 = v_2 = 0.3$, and a plane stress condition is assumed in the analysis. Results of the present method are tabulated in Table II together with those by others [2,7,8] for comparison. It is seen that the differences between those methods are also very small (within 1%).

IV. CONCLUSIONS

In this paper, the theory of the crack tip stress method was extended to the interface crack case with using the relative crack displacements behind the crack tip. Two numerical examples were computed to investigate the accuracy. By using the current method, the results were in very good agreement with the published data. Moreover, the FE mode configuration didn't need very refined mesh around the crack tip in the current procedure, and the computational efficiency was significantly improved.

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TABLE II. NORMALIZED SIFS OF THE EDGE INTERFACE CRACK

SIFs	Central interface crack						
	a/W	Present	Ref[2]	Ref[7]	Ref[8]		
FI	0.1	1.209	1.207	1.201	1.209		
	0.2	1.368	1.365	1.387	1.368		
	0.3	1.653	1.644	1.653	1.654		
	0.4	2.100	2.093	2.100	2.101		
	0.5	2.805	2.791	2.807	2.807		
	0.6	3.998	(3.977)	4.000	4.006		
	0.7	6.284	(6.235)	6.298	6.304		
	0.8	11.768	(11.610)	11.785	11.820		
	0.9	33.735	(32.741)	-	-		
FII	0.1	-0.239	-0.240	-0.238	-0.239		
	0.2	-0.251	-0.251	-0.254	-0.250		
	0.3	-0.288	-0.286	-0.288	-0.288		
	0.4	-0.359	-0.359	-0.359	-0.359		
	0.5	-0.484	-0.484	-0.483	-0.483		
	0.6	-0.716	(-0.718)	-0.701	-0.716		
	0.7	-1.208	(-1.212)	-1.209	-1.208		
	0.8	-2.532	(-2.538)	-2.534	-2.538		
	0.9	-8.797	(-8.742)	-	-		

REFERENCES

- T. Teranishi and H. Nisitani, "Determination of highly accurate values of stress intensity factor in a plate of arbitrary form by FEM," Trans JSME, Series A, vol. 65(635), pp.16-21, 1999.
- [2] K. Oda, K. Kamisugi and N.A. Noda, , "Analysis of stress intensity factor for interface cracks based on proportional method," Trans JSME. Series A, Vol. 75, No.752, pp.476-482, 2009.
- [3] X. Lan, N.A. Noda, K. Michinaka and Y. Zhang, "The effect of material combinations and relative crack size to the stress intensity factors at the crack tip of a bi-material bonded strip," Engng Fract Mech. vol. 78, pp. 2572-2584, 2011.
- [4] X. Lan, N.A. Noda, Y. Zhang and K. Michinaka, "Single and double edge interface crack solutions under arbitrary material combinations," Acta Mechanica Solida Sinica. vol. 25, pp. 404-416, 2012.
- [5] N.A Noda and X. Lan, "Stress intensity factors for an edge interface crack in a bonded semi-infinite plate for arbitrary material combination," Int J. Solids Struct. Vol.49, pp. 1241-1251, 2012.
- [6] MSC Marc 2007r1,2007. MSC Marc 2007r1 user's Guide, 2007. MSC.Software Corp., California, USA
- [7] R. Yuuki, S.B. Cho, Efficient boundary element analysis of stress intensity factors for interface cracks in dissimilar materials. Eng Frac Mech. vol.34:179-188,1989.
- [8] N. Miyazaki, T. Ikeda, T. Soda and T. Munakata, Stress intensity factor analysis of interface crack using boundary element method— Application of contour-integral method. Engng Fract Mech. vol.45(5): 599-610, 1993.

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